A prolongation operator for the Multiscale Finite Element Method coupled with DPG

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The Multiscale Finite Element Method (MsFEM) [1,2] is one of the promising methods used for modeling of heterogeneous materials. It neither requires the assumption of scale separation nor periodicity of microstructure. Moreover, it may be easily parallelized, since the essence of MsFEM is an evaluation of special macroscale finite element trial (approximating) shape functions that capture the microscale details and these functions are computed independently in appropriate patches of elements. Although the method significantly reduces the computational cost, its efficiency and trustworthiness need further improvements. We propose to achieve this goal by coupling MsFEM with the Discontinuous Petrov-Galerkin (DPG) [3] and the ultra weak problem formulation. The important part of MsFEM upscaling is a construction of the macro to micro scale grid prolongation operator that similarly as for the multigrid iterative solvers for heterogeneous materials [4] has to be constructed by the solution of certain auxiliary local boundary value problems in order to incorporate the fine scale properties to the macroscale model. The difficulty of such an operator construction arises here not only due to the material heterogeneity but additionally due to the mixed approximation. Preliminary numerical tests confirm the correctness of the proposed prolongation method.

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